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Algebra 1

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Quiz #3

Justify all your answers completely (Or with a proof or with a counter example) unless mentioned differently.

Problems: We define $Aut(G) = \{\phi : G \to G | \phi \text{ is an isomorphism}\}$ the set of the isomorphism from G onto G and define

$$In(G) = \{f_a : G \to G \text{ defined as } f_a(g) = aga^{-1},\ for\ some\ a \in G\}.$$

Be careful the element of $Aut(G)$ are isomorphisms, not element of G!!!!

1. Show that $(Aut(G), \circ)$ is a group where $\circ$ denote the composition of homomorphism.
2. Show that $(In(G), \circ)$ is a subgroup of $(Aut(G), \circ)$.
3. Let $\phi \in Aut(G)$ and $f_a \in In(G)$, that is there is a a such that $f_a(g) = aga^{-1}$. Express if possible $\phi \circ f_a \circ \phi^{-1}$ as a f_b for some $b \in G$ (Hint: you should evaluate the homomorphism $\phi \circ f_a \circ \phi^{-1}$ at $g \in G$). Deduce that $(In(G), \circ)$ is a normal subgroup of $(Aut(G), \circ)$.
4. Describe $In(G)$, when G is abelian. (Hint: Evaluate an element of $In(G)$ at some element of G, and conclude.).
5. Let $\Psi : G \to Aut(G)$ sending $a \in G$ to the isomorphism $f_a : G \to G$ sending g to aga^{-1}.
   (a) Prove Ψ is an homomorphism. (Hint: Evaluate the image at some $g \in G$).
   (b) Give the definition of $ker(\Psi)$ and $Z(G)$. Prove that $ker(\Psi) = Z(G)$. (Hint: Evaluate the image at some $g \in G$).
   (c) Give the definition of $Range(\Psi)$. Prove that $Range(\Psi) = In(G)$.